

TEMPORAL EVOLUTION OF f -MODE FREQUENCIES AND RADIUS

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ABSTRACT

We have analysed temporal evolution in centroid frequencies and splitting coefficients of solar f -modes obtained from MDI/SOHO. The data were divided into 20 sets covering a period from May 1, 1996 to August 31, 2000. The variation in frequencies is estimated to be 68 nHz over the period of four years which includes the rapidly rising phase of the solar cycle 23. This change is much smaller than that observed for p -mode frequencies. It is also noticed that the f -mode frequencies appear to be weakly correlated with solar activity indices as compared to the p -mode frequencies. We have also inferred the relative change in the solar radii and notice a 1 year periodicity which may be associated with the solar cycle variation.

Key words: Sun: activity—Sun: oscillation.

1. INTRODUCTION

With the availability of precise frequencies from both Global Oscillation Network Group (GONG) and Solar Heliospheric Observatory (SOHO) instruments, refined analysis of solar cycle dependent parameters are being carried out. In these studies, p -mode frequencies have been widely used since these modes penetrate deep inside the sun and thus provide a way to infer changes beneath the surface. In contrast, f -modes, which are the surface gravity modes, are less well studied. Most of the efforts in f -mode studies have been concentrated in estimating the seismic radius of the sun since these mode frequencies are independent of the stratification in the solar interior (Dziembowski et al., 1998, 2000; Antia et al., 2000a). Frequency splittings obtained from f -modes have also been used to study the zonal flows which are associated with torsional oscillations (Howard & LaBonte, 1980) as seen in BBSO data (Woodard & Libbrecht, 1993). These sub-surface flows were first noticed by Kosovichev & Schou (1997) in the Michelson Doppler Imager (MDI) f -mode splittings. This analysis was later extended by Schou (1999) for a longer time period which showed average drift of the

zonal flows towards the equator. Toomre et al. (2000) have confirmed these zonal flows using both p - and f -mode splittings derived from MDI data.

Clearly, we are still lacking a basic understanding as to what causes the frequency changes. Keeping this in view, we analyse the f -mode frequencies and splitting coefficients obtained from the MDI medium- ℓ program for the period which includes the rising phase of the solar cycle 23. We look for possible solar cycle changes in the mean frequencies and even order splitting coefficients. These are further correlated with activity indices. We have also estimated the variations in the seismic radius for the period covered in this study.

2. THE DATA

The results presented here consist of twenty 72 day data sets obtained with the MDI instrument on board SOHO and covers a period from May 1, 1996 to August 31, 2000 with two breakdowns in between (see Schou, 1999 for more details). Each data set consists of centroid frequencies $\nu_{n,l}$ and 36 splitting coefficients a_k . For the present work, only f -modes are used which samples a region very close to the surface. In particular, the analysis is restricted to 41 common modes in the degree range of $217 \leq \ell \leq 286$ and frequency range of $1484 \mu\text{Hz} \leq \nu \leq 1702 \mu\text{Hz}$ unless otherwise mentioned.

3. ANALYSIS AND RESULTS

3.1. Variation of the Sun's Eigen Frequencies

The variation of ℓ averaged frequency shift as a function of frequency for low (May 1 – July 7, 1996) and high (June 21 – August 31, 2000) activity period with reference to set 6 (April 26 – July 6, 1997) is shown in Figure 1. During the minimum activity period, $\delta\nu$ seems to be independent of ν while during high activity period, the frequency shift increases sharply with frequencies. The mean shift $\delta\nu$ for a given ℓ is

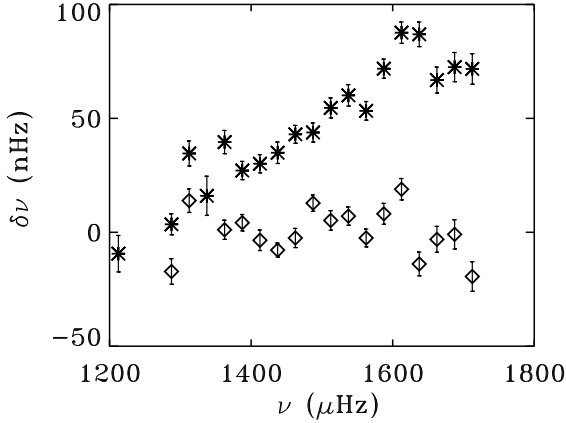


Figure 1. The binned frequency shifts for two independent periods: set 1 (May 1 – July 7, 1996) represented by diamonds and set 20 (June 21 – August 31, 2000) by stars. The shift has been calculated with reference to set 6 (April 26 – July 6, 1997). The error bars represent mean error in shift.

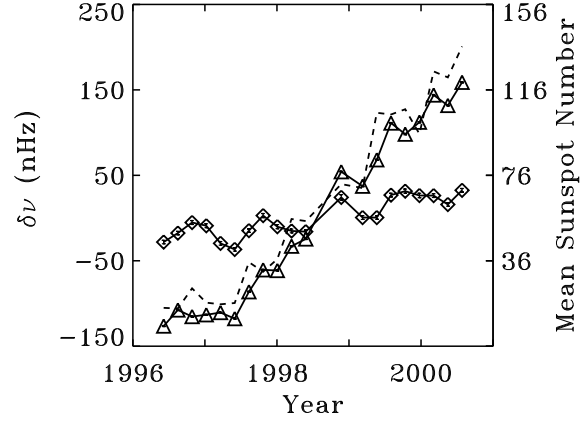


Figure 2. The temporal evolution of frequency shifts for p - (line joined by triangles) and f -modes (line connected by diamonds). The shift has been calculated with respect to mean of all 20 data sets separately. The dashed line represents the solar activity levels denoted here by the International sunspot number. For clarity, the error bars are not shown.

calculated from the relation

$$\delta\nu(t) = \frac{\sum_{\ell} \frac{\delta\nu_{\ell}(t)}{\sigma_{\ell}^2}}{\sum_{\ell} \frac{1}{\sigma_{\ell}^2}} \quad (1)$$

where σ_{ℓ} is the error in the frequency measurement. The variation of $\delta\nu$ with time is shown in Figure 2. Here we notice an oscillatory behaviour with a periodicity of about one year. A similar result is also found by Antia et al. (2000b). The temporal evolution of p -mode frequencies ($1500 \mu\text{Hz} \leq \nu \leq 3500 \mu\text{Hz}$) and the activity index represented by the International sunspot number R_I is also shown in the same figure. It is evident that the change in p -mode frequencies is larger than that of f -mode frequencies. It is further clear that the p -mode frequency shifts follow the activity index quite closely while the mean frequency shifts of f -modes appear to be weakly correlated (also see Table 1).

To study the correlation between mean frequency shifts and solar activity cycle, we have considered the following different activity indices: R_I , the International sunspot number obtained from the Solar Geophysical Data (SGD); KPMI, Kitt Peak Magnetic Index which represents the line of sight magnetic field strength and averaged over the full disk from Kitt Peak magnetograms; MPSI, Magnetic Plage Strength Index from Mount Wilson magnetograms which represents the absolute values of the magnetic field strengths between 10 and 100 gauss; MWSI, Mount Wilson Sunspot Index which represents the magnetic field strengths above 100 gauss; He I, equivalent width of He I 10830 Å line, averaged over the whole disk from Kitt peak and F_{10} , integrated radio flux at 10.7 cm from SGD. Both Pearson's (r_p) and Spearman's (r_s) correlation coefficients are given in Table 1.

Table 1. Correlation statistics for f - and p -mode frequency shifts.

Activity Index	f -mode		p -mode	
	r_p	r_s	r_p	r_s
R_I	0.84	0.80	0.98	0.93
KPMI	0.88	0.78	0.99	0.94
MPSI	0.87	0.77	0.99	0.95
MWSI	0.87	0.82	0.91	0.83
F_{10}	0.87	0.78	0.99	0.96
He I	0.85	0.79	0.99	0.97

It is observed that in case of f -modes, the activity indices have weak correlation with frequency shifts. In comparison, p -modes display higher correlation coefficients showing stronger correlation with activity indices as was earlier pointed out by Jain, Tripathy, & Bhatnagar (2000) and Howe, Komm, & Hill (1999). However, it is interesting to note that MWSI with other magnetic indices has strong correlation with f -modes while it has the weakest correlation with p -modes. Thus, it appears that MWSI which represents the average magnetic field strength above 100 gauss behaves differently with p - and f -modes and needs to be investigated more closely as the solar cycle progresses. The observation that f -modes in comparison with p -modes are weakly correlated with magnetic field indices further confirms that magnetic fields are present at deeper layers rather than near the solar surface.

3.2. Seismic Radius

Since the early nineteen century, the temporal variation in the Sun's diameter has been reported by many observers (Lacare et al., 1996; Noel, 1997). The recent measurements report a change of 0.1 arcsec to 1 arcsec in the measured semi-diameter which implies a change of 70 to 700 km in the radius. Assuming that the change in frequencies is entirely due to the change in radius and there is no contribution from the magnetic field, convection etc., the average frequency difference is related to change in radius by

$$\left\langle \frac{\delta\nu}{\nu} \right\rangle = -\frac{3}{2} \frac{\delta R}{R}, \quad (2)$$

where $\delta\nu$ is the centroid frequencies calculated earlier and the differences are taken with respect to the mean of all 20 data sets. The evolution of the relative differences in the seismic radius is shown in Figure 3. The maximum change is found to be 29.8 km over a period of four years, the seismic radius decreasing with increase of activity levels and thus indicating anti-correlation between them. A relative variation in the solar radius of the order of 6×10^{-6} was first reported by Dziembowski et al. (1998) which corresponds to approximately 4 km over a period of 2 years around the solar minimum. If this variation is correlated with solar activity, we would expect much larger change, as reported in this paper, between minimum and maximum activity period. However, the analysis of Dziembowski et al. (2000) did not find any systematic trend of correlation with sunspot number. A similar study using GONG data (Antia et al., 2000a) for the period 1995 to 1998 reported a decrease in solar radius by about 5 km, during the period in which the solar activity was increasing. It has been known that the high wave number modes are known to deviate significantly from the simple dispersion relation (Duvall, Kosovichev & Murawski, 1998; Antia & Basu, 1999) and the inclusion of higher degree in our analysis may cause this large change in the solar radius.

In order to check the stability of our results, we have repeated the analysis by considering the frequency tables which contains the splitting coefficients only up to a_6 and thus lowering the ℓ values to 119. The relative difference in radius in this case is found to be 14 km and clearly demonstrates the degree dependence. We have also estimated the relative difference in seismic radius from activity minimum to maximum by restricting the common modes between $100 \leq \ell \leq 200$ and this is shown as dashed line in Figure 3. In this case, the change in radius is found to be 7 km, consistent with the results of Antia et al. (2000a) which used the same range of ℓ values.

Since the change in seismic radius reflects the relative differences in frequencies, the oscillatory pattern visible in Figure 2 persists in Figure 3. However, it is not clear if this period is associated with solar cycle variations since the data used here spans only 4 years. Also, this period is very close to the orbital

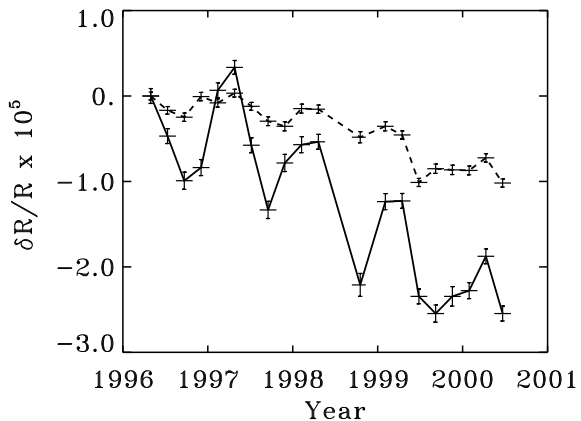


Figure 3. Relative differences in the radius of the sun as inferred from f-mode frequencies. The solid line is the result as derived from frequency tables containing splitting coefficients up to a_{36} and covers the degree range of 217 to 286. The dashed line shows the result using the frequency tables containing splitting coefficients up to a_6 and includes modes of $100 \leq \ell \leq 200$ only.

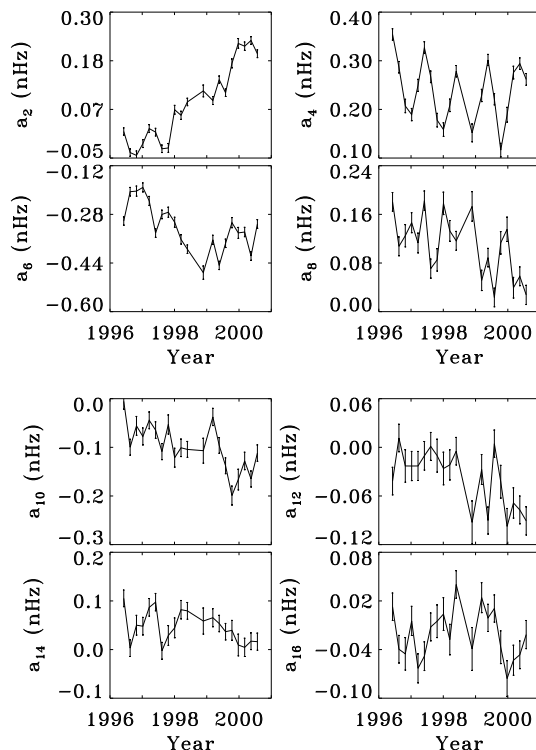


Figure 4. Average even order splitting coefficients as a function of index and time. For each index i , the a -coefficients are joined with lines denoting temporal evolution. Error bars are 1σ . The averaging was done only over common modes.

period of the earth and one may expect systematic errors of this period being introduced in the data.

3.3. Even Order Splitting Coefficients

The variation of even order coefficients up to a_{16} for f -modes are shown in Figure 4. These coefficients seem to be uncorrelated with solar cycle except for a_2 . In contrast, the even order splitting coefficients of p -modes are known to be correlated with solar cycle. In particular, Howe, Komm, & Hill (1999) find significant correlation between the even a -coefficients and the corresponding Legendre coefficients of a fit to the Kitt Peak magnetograms. A similar result based on the BBSO Ca K plage data for higher order was recently reported by Dziembowski et al. (2000).

4. CONCLUSIONS

We have analysed the f -mode frequencies and splitting coefficients obtained from MDI medium- ℓ program for the period May 1996 to August 2000. We find that the mean shift in these mode frequencies are weakly correlated with activity indices as compared to the p -mode frequencies. In case of MWSI, which represents the magnetic field above 100 Gauss, the f -modes have the best correlation while in case of p -modes, the correlation is weakest. We have also estimated the relative difference in the seismic radius as due to the change in frequencies and find that the change is a function of degree of the mode. If we consider only those modes with degrees between 100 and 200, we find the change in solar radius from minimum to maximum of the solar cycle is approximately 7 km and is one order smaller than that is expected from the analysis of MDI data by Dziembowski et al. (1998). If we use the high degree modes ℓ up to 286 which is approximately the same as used by Dziembowski et al., we find a significantly higher value. We also report here a 1 year periodicity in both the f -mode frequencies and seismic radius, although it is not clear if these are associated with the solar cycles or reflects the systematic errors in the data.

ACKNOWLEDGMENTS

This work utilises data from the Solar Oscillations Investigation / Michelson Doppler Imager on the Solar and Heliospheric Observatory. We would like to thank J. Schou for providing us the MDI data. SOHO is a mission of international cooperation between ESA and NASA. NSO/Kitt Peak magnetic, and Helium measurements used here are produced cooperatively by NSF/NOAO; NASA/GSFC and NOAA/SEL. This work is partially supported under the CSIR Emeritus Scientist Scheme and Indo-US collaborative programme-NSF Grant INT-9710279.

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